Stationary planar domain walls of a classical spin chain

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Abstract

Domain walls of a discrete model of an anisotropic ferromagnet are studied. They can be described by sequences of two reversible mappings. Competition between the length scale of spatial structures and the lattice constant leads to a rich diversity of domain wall solutions related in a bifurcation scenario.

Key words: reversible mapping, iterated function system, homoclinic orbit
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Introduction

The complexity of spatial structures of discrete oscillator chains raises questions of considerable interest [1]-[8]. In this paper domain wall solutions of an one-dimensional discrete model of a ferromagnet are contrasted to the well-known Bloch wall solution of the corresponding continuous system. As is known [9], stationary planar solutions of

$$\dot{\vec{S}} = \vec{S} \times (J \frac{\partial^2}{\partial x^2} \vec{S} + a S_z \vec{e}_z) \tag{1}$$

are represented by the equation $\phi'' = -\frac{a}{J} \cos\phi \sin\phi$ of a point mass in a periodic potential; beside spatially periodic solutions, the kink solution

$$\sin\phi = \pm \mathrm{tgh} \sqrt{\frac{a}{J}} x \tag{2}$$

is obtained. However, if the length $\sqrt{\frac{J}{a}}$ of this kink is not large enough, the continuum approximation [10] fails and one expects competition between the length scale of a spatial structure and the lattice constant. Several papers have studied the corresponding classical discrete system [1], [2].

This paper is devoted to study of stationary localized structures connecting two homogeneous domains in a model of a ferromagnetic chain with one kind of atom; quantum corrections [3] are neglected.

Stationary solutions of oscillator chains with appropriate next neighbour coupling can be described by mappings; e.g. the Frenkel-Kontorova system is associated to the standard map [4],[5]. In contrast, complexity of spin chains arises from the fact that the coordinates of one spin oscillator are not given unambiguously by the position of its two predecessors. For planar solutions two positions of the following spin are possible. Consequently only a small part of the stationary states can be reached by a single map. At every single step one can decide which one of two possible maps is to be applied, so that the system may be considered an explicitly time-dependent dynamical system. Thus, according to symbolic dynamics, a subdivision of the solutions with their defining symbol sequence suggests itself. Solutions which are characterized by the same inversion symmetry as the Bloch wall are related to symmetry lines of the mappings.

The anisotropic spin chain

We discuss stationary planar solutions of the model of a spin chain [6],[7],[8] with isotropic ferromagnetic coupling and an uniaxial anisotropic term

$$\dot{\vec{S}_n} = \vec{S}_n \times (J(\vec{S}_{n+1} + \vec{S}_{n-1}) + a S_{nz} \vec{e}_z) \tag{3}$$

J is the isotropic Heisenberg coupling constant, a an anisotropy constant; these constants are real. The modulus of the spin is conserved, we set $|\vec{S}_n| = 1$. $\vec{S}_n$ is fixed if the local effective field $\vec{h}_{eff\,n} = J(\vec{S}_{n-1} + \vec{S}_{n+1}) + a S_{nz} \vec{e}_z$ is parallel to $\vec{S}_n$. The position of $\vec{S}_{n+1}$ can be computed from the positions of $\vec{S}_n$ and $\vec{S}_{n-1}$. This determination is not unique;

in general several directions $\vec{S}_{n+1}$ yield parallel effective fields $\vec{h}_{effn}$. In the same way, possible directions of $\vec{S}_{n-2}$ can be computed.
The system is rotational symmetric with regard to the z-axis. Introducing spherical coordinates

$$\vec{S}_n = (\cos\theta_n, \sin\theta_n \sin\phi_n, \sin\theta_n \cos\phi_n) \tag{4}$$

we investigate solutions with spins perpendicular to the x-axis ($\theta = \frac{\pi}{2}$). In this case, fixed point conditions are given by the system of equations

$$\sin(\phi_{n+1} - \phi_n) = \sin(\phi_n - \phi_{n-1}) + \frac{a}{2J}\sin(2\phi_n) \tag{5}$$

Introducing the second coordinate $I_n = \sin(\phi_n - \phi_{n-1})$ the fixed point condition can be satisfied equally by

$$L_1 : \begin{array}{l} \phi_{n+1} = \phi_n + \arcsin I_{n+1} \\ I_{n+1} = I_n + k \sin 2\phi_n \\ k = \frac{a}{2J} \end{array} \tag{6}$$

and by

$$L_2 : \begin{array}{l} \phi_{n+1} = \phi_n + \pi - \arcsin I_{n+1} \\ I_{n+1} = I_n + k \sin 2\phi_n \\ k = \frac{a}{2J} \end{array} \tag{7}$$

Stationary solutions are defined by their initial conditions and a sequence of L_1's and L_2's, a property reminiscent of symbolic dynamics. In general, there are several solutions with the same sequence but with different initial conditions. In this sense, our system is equivalent to a lowdimensional dynamical system with an explicit time dependence which causes the sequence of mappings L_1, L_2.
I_n must not exceed 1. For this reason, not all initial conditions lead to a stationary solution. The domain of definition is in general a complicated subset of the cylinder $[0,1] \times [0, 2\pi]$.
Each mapping commutes with

$$M^- : \begin{array}{l} \phi' = -\phi \\ I' = -I \end{array} \tag{8}$$

and with

$$M^\pi : \begin{array}{l} \phi' = \phi + \pi \\ I' = I \end{array} \tag{9}$$

L_1 and L_2 are reversible mappings [11], i.e. they can be decomposed into involutions $L = H \circ G$ with $H^2 = G^2 = Id$ (identity mapping). It follows that $L^{-1} = G \circ H$. Employing M^- and M^π they can each be decomposed into involutions in four different ways:

$$L_1 = H_1^{0+} \circ G^{0+} = H_1^{\pi+} \circ G^{\pi+} = H_1^{0-} \circ G^{0-} = H_1^{\pi-} \circ G^{\pi-} \tag{10}$$

$$L_2 = H_2^{0+} \circ G^{0+} = H_2^{\pi+} \circ G^{\pi+} = H_2^{0-} \circ G^{0-} = H_2^{\pi-} \circ G^{\pi-} \tag{11}$$

We draw up the involutions and symmetry lines $Fix(G), Fix(H)$:

involution	symmetry lines
$H_1^{0+}: \begin{array}{l} \phi' = -\phi + \arcsin I \\ I' = I \end{array}$	$\phi = \frac{\arcsin I}{2}$
$G^{0+}: \begin{array}{l} \phi' = -\phi \\ I' = I + k \sin 2\phi \end{array}$	$\phi = 0$
$H_1^{\pi+}: \begin{array}{l} \phi' = \pi - \phi + \arcsin I \\ I' = I \end{array}$	$\phi = \frac{\pi + \arcsin I}{2}$
$G^{\pi+}: \begin{array}{l} \phi' = \pi - \phi \\ I' = I + k \sin 2\phi \end{array}$	$\phi = \frac{\pi}{2}$
$H_1^{0-}: \begin{array}{l} \phi' = \phi - \arcsin I \\ I' = -I \end{array}$	$I = 0$
$G^{0-}: \begin{array}{l} \phi' = \phi \\ I' = -I - k \sin 2\phi \end{array}$	$I = -\frac{k \sin 2\phi}{2}$
$H_1^{\pi-}: \begin{array}{l} \phi' = \phi - \arcsin I + \pi \\ I' = -I \end{array}$	$-$
$G^{\pi-}: \begin{array}{l} \phi' = \phi + \pi \\ I' = -I - k \sin 2\phi \end{array}$	$-$
$H_2^{0+}: \begin{array}{l} \phi' = -\phi + \pi - \arcsin I \\ I' = I \end{array}$	$\phi = \frac{\pi - \arcsin I}{2}$
$H_2^{0-}: \begin{array}{l} \phi' = \phi - \pi + \arcsin I \\ I' = -I \end{array}$	$-$
$H_2^{\pi+}: \begin{array}{l} \phi' = -\phi + 2\pi - \arcsin I \\ I' = I \end{array}$	$\phi = \pi - \frac{\arcsin I}{2}$
$H_2^{\pi-}: \begin{array}{l} \phi' = \phi + \arcsin I \\ I' = -I \end{array}$	$I = 0$

(12)

The fact that L_1 and L_2 have the same involutions $G^{0/\pi\pm}$ has a consequence of importance in the further argumentation. We assume U to be any finite sequence of L_1's and L_2's and V the corresponding sequence with reversed order of indices, for instance $U = L_2 \circ L_2 \circ L_1 \circ L_2$ and $V = L_2 \circ L_1 \circ L_2 \circ L_2$. It follows that

$$U^{-1} = G \circ V \circ G \tag{13}$$

(we skip the indices of G and H if they are unimportant).

Continuous families of domain walls

In this chapter symmetry properties of domain wall structures are studied. In our context we understand a domain as a homogeneous or spatially periodic structure. A domain can be defined as a fixed point of a finite sequence $D = {L_1}^\alpha \circ {L_2}^\beta \circ {L_1}^\gamma ...$. For example, $(I = 0, \phi = 0)$ is a fixed point of $D = L_1$. This corresponds to the ferromagnetic state with parallel spins. $(I = 0, \phi = 0)$ is also a fixed point of $D = {L_2}^2$. In this case it corresponds to the antiferromagnetic state of antiparallel spins.

Domain walls are stationary structures connecting two homogeneous domains. Let D_1, D_2 denote two finite sequences of L_1 and L_2 which characterize the domains. (I_{D_1}, ϕ_{D_1}), (I_{D_2}, ϕ_{D_2}) are fixed points of D_1 and D_2 respectively. A domain boundary solution connecting the two saddle points (I_{D_1}, ϕ_{D_1}), (I_{D_2}, ϕ_{D_2}) may contain a sequence of mappings different from D_1 and D_2, so a domain boundary solution is defined as a point set determined by the sequence $\bar{D}_1 \circ B \circ \bar{D}_2$. Here $\bar{D}_1$ and $\bar{D}_2$ denote infinite sequences of D_1 and D_2 while B is an arbitrary finite sequence of L_1 and L_2. Such a domain boundary solution is obtained if one considers a point sequence emanating from a fixed point of D_1 on its unstable manifold $W^u_{D_1}$. After the application of a certain sequence B of the mappings L_1 and L_2 the point sequence must follow again the stable manifold $W^s_{D_2}$ towards a fixed point of D_2; consequently domain walls are described as heteroclinic orbits of the saddle points. In correspondence with the solution of the continuous system, we are particularly interested in inversion symmetrical solutions. Such solutions can only be obtained if D_1 and D_2 are defined as sequences with reversed order of indices, for example $D_1 = L_1 \circ (L_2)^3, D_2 = (L_2)^3 \circ L_1$ or $D_1 = D_2 = L_2$. B can be an arbitrary finite symmetric sequence, for instance $(L_2)^2$ or ${L_2}^2 \circ L_1 \circ {L_2}^2 \circ L_1 \circ {L_2}^2$. (I_0, ϕ_0) is a point on the unstable manifold $W^u_{D_1}$ of the saddle point $(I_{D_1}, \phi_{D_1}) = L_1^{-\infty}(I_0, \phi_0)$.

$$S^u_{D_1}(I_0, \phi_0) = \{...(I_0, \phi_0), (I_1, \phi_1), ...\} \subset W^u_{D_1} \tag{14}$$

is the set of all images $D_1^n(I_0, \phi_0), n \in Z$ of (I_0, ϕ_0). Applying an involution G to this set and using $D_2^{-1} = G \circ D_1 \circ G$ (13), $G^2 = Id$

$$S^s_{D_2} = G(S^u_{D_1}(I_0, \phi_0)) = \{...G(I_0, \phi_0), G(I_1, \phi_1), ...\} \tag{15}$$

turns out to be the set of iterations $({D_2}^{-1})^n(G(I_0, \phi_0))$, which is an orbit on the stable manifold $W^s_{D_2}$ of the saddle point $G(I_{D_1}, \phi_{D_1})$. So the involution G transforms the unstable manifold of a fixed point into the stable manifold of another fixed point.

The sets $S^u_{D_1}$ and $S^s_{D_2}$ represent a heteroclinic orbit if they are identical. This is true if a common point of $S^u_{D_1}$ and $S^s_{D_2}$ exists, e.g. a fixed point of an involution. $S^u_{D_1}$ or $S^s_{D_2}$ represent an intersection point set of $W^u_{D_1}$ and $W^s_{D_2}$. This is the most simple case without a central sequence B different from D_1 or D_2.

For a domain wall associated to a sequence B, a point $(I_B, \phi_B) \in W^u_{D_1}$ must exist which is mapped by B onto $W^s_{D_2}$. So we must find an intersection point of $B(W^u_{D_1})$ and $W^s_{D_2}$ in order to obtain a domain wall solution. B must cause a jump from a point of $S^u_{D_1}$ to the corresponding point of $S^s_{D_2}$; i.e. for one point (I_B, ϕ_B) of $W^u_{D_1}$ the conditions

$$B(I_B, \phi_B) = G(I_B, \phi_B) \tag{16}$$

must be fulfilled. We will show that sequences $\bar{D}_1 \circ B \circ \bar{D}_2$ lead to structurally stable domain wall solutions. There are three different forms of the central symmetric sequence B:

$$B = U \circ V \tag{17}$$

$$B = U \circ L_1 \circ V \tag{18}$$

$$B = U \circ L_2 \circ V \tag{19}$$

where U and V have inverse order of indices (13). We will show that any intersection point of (17) $V(W^u)$ or (18) $L_1 \circ V(W^u)$ or (19) $L_2 \circ V(W^u)$ and a symmetry line of (17) G, (18)

H_1 or (19) H_2 yields an inversion symmetric solution. So to speak, we just have to find an intersection point of two lines on a plane; no additional requirement must be satisfied for reversibility.
For a symmetric solution we have to find a point (I_B, ϕ_B) of W^u with the property (16) We focus the case (17) $B = U \circ V$. It will be shown that condition (16) follows from the existence of an intersection point $(I_C, \phi_C) = V(I_B, \phi_B)$ of $V(W^u_{D_1})$ and a symmetry line of $G^{0/\pi\pm}$. Clearly such a point is invariant under the G concerned. With the property (13) $V^{-1} = G \circ U \circ G$ we have

$$(I_B, \phi_B) = V^{-1}(I_C, \phi_C) = G \circ U \circ G(I_C, \phi_C) \tag{20}$$

Applying G and using $G^2 = Id$, $G(I_C, \phi_C) = (I_C, \phi_C)$ we obtain

$$G(I_B, \phi_B) = U(I_C, \phi_C) = U \circ V(I_B, \phi_B) = B(I_B, \phi_B) \tag{21}$$

which had to be proved.
For the case (18) $B = U \circ L_1 \circ V$ we have to prove

$$U \circ L_1 \circ V(I_B, \phi_B) = G^{0/\pi\pm}(I_B, \phi_B) \tag{22}$$

where (I_B, ϕ_B) is a point on $W^u_{D_1}$. Presupposing an intersection point $(I_C, \phi_C) = L_1 \circ V(I_B, \phi_B)$ of a symmetry line of $H_1^{0/\pi\pm}$ and the line $L_1 \circ V(W^u)$, we see that

$$(I_C, \phi_C) = H_1(I_C, \phi_C) = H_1 \circ L_1(I_{C-1}, \phi_{C-1}) = G(I_{C-1}, \phi_{C-1}) \tag{23}$$

and consequently

$$(I_B, \phi_B) = V^{-1}(I_{C-1}, \phi_{C-1}) = G \circ U \circ G(I_{C-1}, \phi_{C-1}) \tag{24}$$

Applying G we obtain

$$G(I_B, \phi_B) = U(I_C, \phi_C) = U \circ L_1 \circ V(I_B, \phi_B) \tag{25}$$

This can be carried out in the same way for (19) $B = U \circ L_2 \circ V$, in which case we presuppose an intersection point of a symmetry line of $H_2^{0/\pi\pm}$ and the line $L_2 \circ V(W^u_{D_1})$. We conclude that symmetric solutions constitute a continuous family with respect to the parameter k. This proof does not imply that all solutions are symmetric. Intersection points of $B(W^u_{D_1})$ and $W^s_{D_2}$ may exist which do not correspond to any symmetry line.

Bifurcations of domain walls

To put this concept in concrete terms, we focus on solutions which are similar to the Bloch wall solution, i.e. inversion symmetrical solutions with $D_1 = D_2 = L_1$ belonging to the fixed points $(0, 0)$ and $(0, \pi)$ of L_1. Applying the above considerations we identify the solutions by intersection points of $W^u_{D_1}$ or $U(W^u_{D_1})$ and symmetry lines.
Symmetric domain wall solutions can be divided into two classes [2]: Configurations with the inversion symmetry point on a lattice site are described as central-spin solutions. They correspond to central sequences of $B = U \circ V$ (17).

The remaining (central-bond) configurations are characterized by an inversion symmetry point in the middle between two lattice sites. They correspond to $B = U \circ L_1 \circ V$ (18) or to $B = U \circ L_2 \circ V$ (19)
In the following, selected solutions described by central sequences $B = L_1$, L_1^2 , L_2 or L_2^2 are studied.
Typical solutions for moderate parameter values ($k = \frac{a}{2J} = 0.28$) can be inferred from figure 1 which shows the sector $[0,1] \times [0,\pi]$ of the phase space. The sector can be continued with the period π by the symmetry M^π, and by M^- it is inversion symmetrical. This corresponds to clockwise and anticlockwise solutions.
An intersection point of $W^u_{L_1}$ and $W^s_{L_1}$ is mapped to the next intersection point but one. Therefore two solutions arise from intersection points of $W^u_{L_1}$ and $W^s_{L_1}$. Corresponding to (21), a heteroclinic set with $B = L_1^2$ exists which contains the intersection point s of $W^u_{L_1}$, $W^s_{L_1}$ and the symmetry line $\phi = \frac{\pi}{2}$ of $G_1^{\pi+}$. This solution contains a central spin with $\phi = \frac{\pi}{2}$ (figure 2).
Corresponding to (25) the heteroclinic set with $B = L_1$ contains an intersection point b of $W^u_{L_1}$, $W^s_{L_1}$ and the symmetry line $\phi = \frac{\pi + \arcsin I}{2}$ of $H_1^{\pi+}$. The center of this solution is in the middle between two lattice sites. $(0,0)$ is an intersection point of $W^u_{L_1}$ and $Fix(H_2^{\pi+})$; so there exists a sudden kink $\phi_i = 0$ for $i < 0$, $\phi_i = \pi$ for $i > 0$. This solution of type $B = L_2$ corresponds to the Bloch wall (2) for $\frac{a}{J} \to \infty$.
A differently shaped domain wall of type $B = L_2^2$ corresponds to the intersection point $PS1$ of ${L_2}^2(W^u_{L_1})$ and $W^s_{L_1}$. The predecessor $S1$ of $PS1$ is an intersection point of $L_2(W^u_{L_1})$ and the symmetry line $\phi = \frac{3\pi}{2}$ of $G_1^{\pi+}$, so the centre of this solution is $\phi = \frac{3\pi}{2}$ (figure 3).
If a localized solution arises from one and the same domain on its right and its left side, it is called hump. We pick out three simple hump solutions represented by homoclinic orbits of the dynamical system. The intersection point C of $L_2(W^u_{L_1})$, $W^s_{L_1}$ and the symmetry line $\frac{\pi - \arcsin I}{2}$ of H_2^{0+} indicates a homoclinic $B = L_2$ solution, which connects $(0,0)$ to itself and is centred at $\phi = \pi$.
There exists also a homoclinic solution of $B = L_2^2$ type which can be seen from the intersection point $PS2$ of ${L_2}^2(W^u_{L_1})$ and $W^s_{L_1}$. The predecessor $S2$ of $PS2$ is an intersection point of $L_2(W^u)$ and the symmetry line $\phi = \pi$ of G^{0+}; the solution is centred at $\phi = \pi$. The origin $(0,0)$ is also an intersection point of these two lines which corresponds to a trivial hump $\phi_i = 0, \phi_0 = \pi, \phi_j = 0, i < 0, j > 0$ where only one spin is folded down in relation to the ferromagnetic state.
In the following, bifurcations of these solutions will be briefly described. To determine the stability of solutions in question the equations of motion for a number (usually 30-50) of spins of the domain wall are linearized while the boundaries are fixed. Eigenvalues of this matrix are computed numerically. Due to the rotation symmetry of the problem, two eigenvalues are zero. The solution corresponds to a local energy minimum if all other eigenvalues are imaginary. This fixed point becomes attractive if a phenomenological Landau-Lifshitz damping term [12] is added.
The solution corresponding to intersection point b is stable. It exists up to $k = 0.53$, where the maximal I of the L_1 solution (point b) reaches 1. L_1 and L_2 have the same effect at this point. It follows that a $B = L_1$ solution is identical with a $B = L_2$ solution. Indeed, with this parameter value a branch of $L_2(W^u_{L_1})$ touches $W^s_{L_1}$. So the $B = L_1$ solution merges into a $B = L_2$ solution with an opening angle larger than $\frac{\pi}{2}$. This solution is stable as well.

For increasing k, the intersection point of $L_2(W^u_{L_1})$ and $W^s_{L_1}$ is moving down to $I = 0, \phi = \pi$. At this point, the clockwise L_2 solution and its anticlockwise counterpart merge into the sudden kink solution in a pitchfork-bifurcation. In this bifurcation the sudden kink solution corresponding to the intersection point $(0, 0)$ of $W^u_{L_1}$ and $Fix(H_2^{\pi+})$ becomes stable. We can compute the parameter value of this bifurcation analytically. The slopes of $W^s_{L_1}$ and of $L_2(W^u_{L_1})$ at $(0, \pi)$ must coincide, i.e.

$$- k - \sqrt{2k + k^2} = \frac{k + \sqrt{2k + k^2}}{1 - k - \sqrt{2k + k^2}} \tag{26}$$

must be satisfied, which happens at $k = \frac{2}{3}$. This is the analytical confirmation of the numerically determined value in [1].
The solution containing a central spin s is stable and has a slightly higher energy. It exists for all parameter values. For $\frac{a}{J} \to \infty$ we get $\phi_i \to 0$ for $i < 0$, $\phi_i = \frac{\pi}{2}$ for $i = 0$ $\phi_i \to \pi$ for $i > 0$.
The solution corresponding to the intersection point $S2$ appears in a pitchfork-bifurcation out of a trivial solution $\phi_i = 0, \phi_0 = \pi, \phi_j = 0, i < 0, j > 0$. At the critical parameter value the gradients of $W^s_{L_1}$ and ${L_2}^2(W^u_{L_1})$ must coincide. The value is computed analogously to (26), we get $k = \frac{1}{4}$. For $k < 1.28$ the trivial hump is unstable, for larger k it is stable. All other solutions of figure 3 are unstable.

Conclusion

In contrast to the related continuous system, the discrete spin chain has a large variety of stationary planar solutions. As a peculiarity of spin systems, a virtual sequence of two mappings generates a stationary solution. Consequently a solution is defined both by its initial conditions and a generating symbol sequence. Besides a variety of solutions with no correlate in the continuous model, the discrete spin chain has several solutions similar to the Bloch wall solution. The latter leads to degeneration if the continuum limit is a good approximation (weak anisotropy): Two stable solutions of the discrete system converge to the same continuum limit. On the other hand, for strong anisotropy, one of these solutions bifurcates into an abrupt kink of 180 degrees. Among the immense number of solutions of a nonlinear mapping, chaotic orbits can be identified with stationary states of the spin chain. For moderate values of k, some KAM-tori have already vanished but others still ensure that the trajectory does not leave the definition area. Because of energetical instability, the physical relevance of these solutions should be assessed with care. Even stable solutions may be unobservable due to their high energy, but also unstable stationary solutions may be relevant in dynamics. Firstly, unstable stationary and periodic solutions contribute to the structure of chaotic attractors and are relevant for their construction [13]. Secondly it was suggested [14] that unstable stationary solutions influence transient dynamics in the process of pattern formation.

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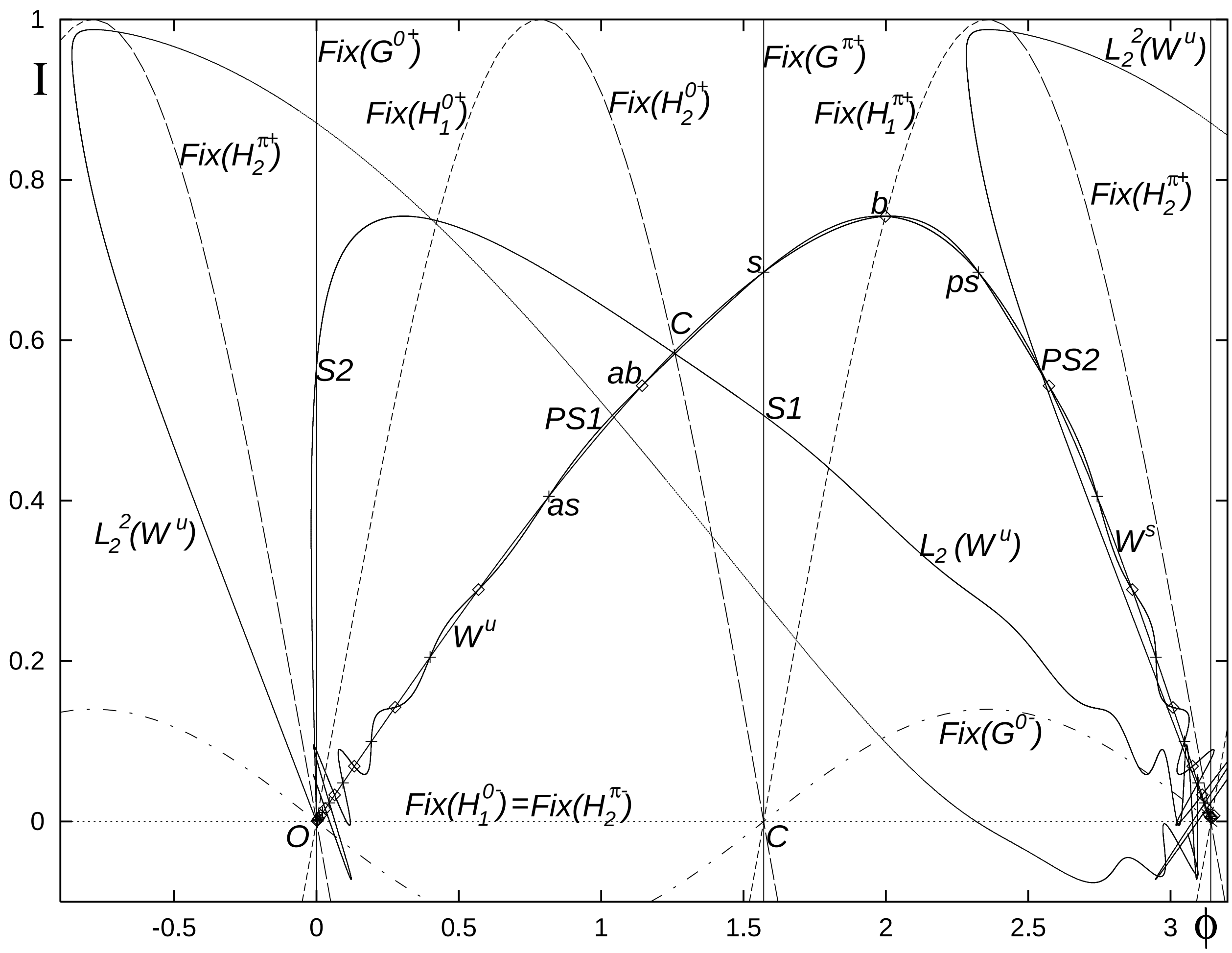
Figure captions

Figure 1
The sector $[0,1]\times[0,\pi]$ of the phase space for $k=0.28$. It shows W^u, W^s, $L_2(W^u)$, ${L_2}^2(W^s)$ and all symmetry lines. Here $L_2(W^u)$ is the image of W^u of the segment $\phi\in[-\pi,0]$. There are a central-bond set of intersection points of W^u and W^s including the intersection point b of W^u, W^s, $Fix(H_2^{\pi+})$ and a central-spin set including the intersection point s of W^u, W^s, $Fix(G^{\pi+})$. The intersection point B of $L_2(W^u)$, W^s and $Fix(H_2^{0+})$ belongs to a central-bond solution. There are two other central-spin solutions: One of them corresponds to the intersection point $PS1$, its predecessor $S1$ is a intersection point of $L_2(W^u)$ and the symmetry line $\phi=\pi$. The predecessor $S2$ of the point $PS2$ is an intersection point of $L_2(W^u)$ and $\phi=0$. The origin of this solution is a pitchfork bifurcation at $k=0.25$ where the gradients of W^s and ${L_2}^2(W^u)$ at $(0,\pi)$ coincide.

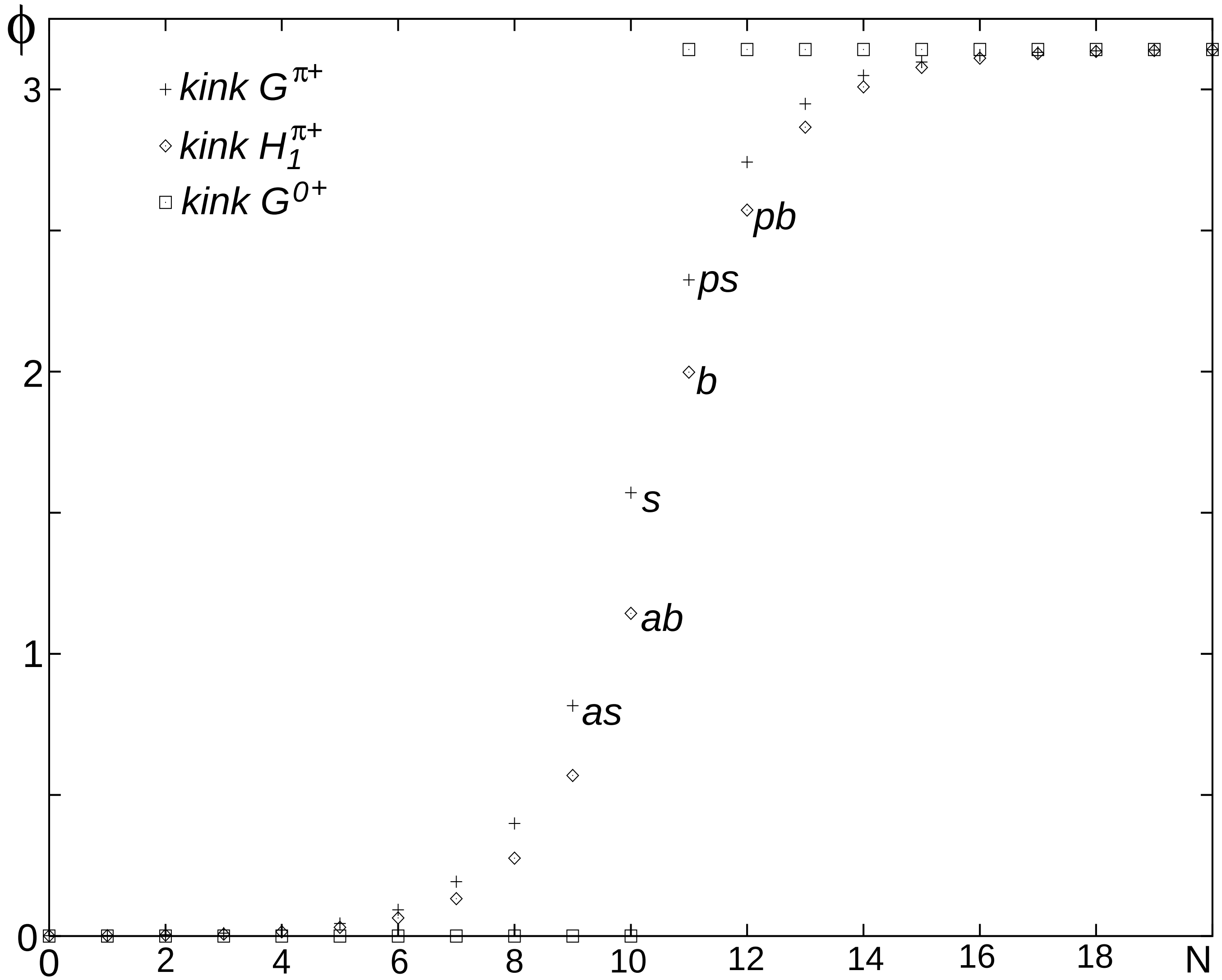
Figure 2
The angle ϕ of three basic domain wall solutions as a function of the spin index N at $k=0.28$. The kinks corresponding to symmetry lines $G^{\pi+}$ and $H_1^{\pi+}$ are energetically stable, these solutions are generated only by L_1. The sudden kink corresponding to the symmetry line G^{0+} is caused by a central application of $B=L_2$. It becomes stable at $k=\frac{2}{3}$ (weak coupling and virtually isolated spins).

Figure 3
The domains $\phi=0$ and $\phi=\pi$ are linked by a solution corresponding to the symmetry line $G^{\pi+}$ (note ϕ mod2π). Discernible from opening angles larger than $\frac{\pi}{2}$, this solution contains a central sequence $B={L_2}^2$. So $PS1$ is generated from its predecessor $S1$ by L_2. Similarly, the hump G^{0+} contains $B={L_2}^2$, while the hump H_2^{0+} contains $B=L_2$. At the trivial hump G^{0+}, only one spin $S2$ is folded down in relation to the ferromagnetic domain. This solution becomes stable for weak coupling ($k=1.28$) while all other solutions in this picture are unstable.



I
Φ
Fix(G^0+)
Fix(G^π+)
Fix(H_1^0+)
Fix(H_2^0+)
Fix(H_1^π+)
Fix(H_2^π+)
L_2^2(W^u)
Fix(H_2^π+)
b
s
ps
C
PS2
ab
S2
PS1
S1
as
L_2^2(W^u)
L_2 (W^u)
W^s
W^u
Fix(G^0-)
Fix(H_1^0-)=Fix(H_2^π-)
O
C

figure 1



φ
kink $G^{\pi+}$
kink $H_1^{\pi+}$
kink G^{0+}
pb
ps
b
s
ab
as
0
1
2
3
0
2
4
6
8
10
12
14
16
18
N

figure 2

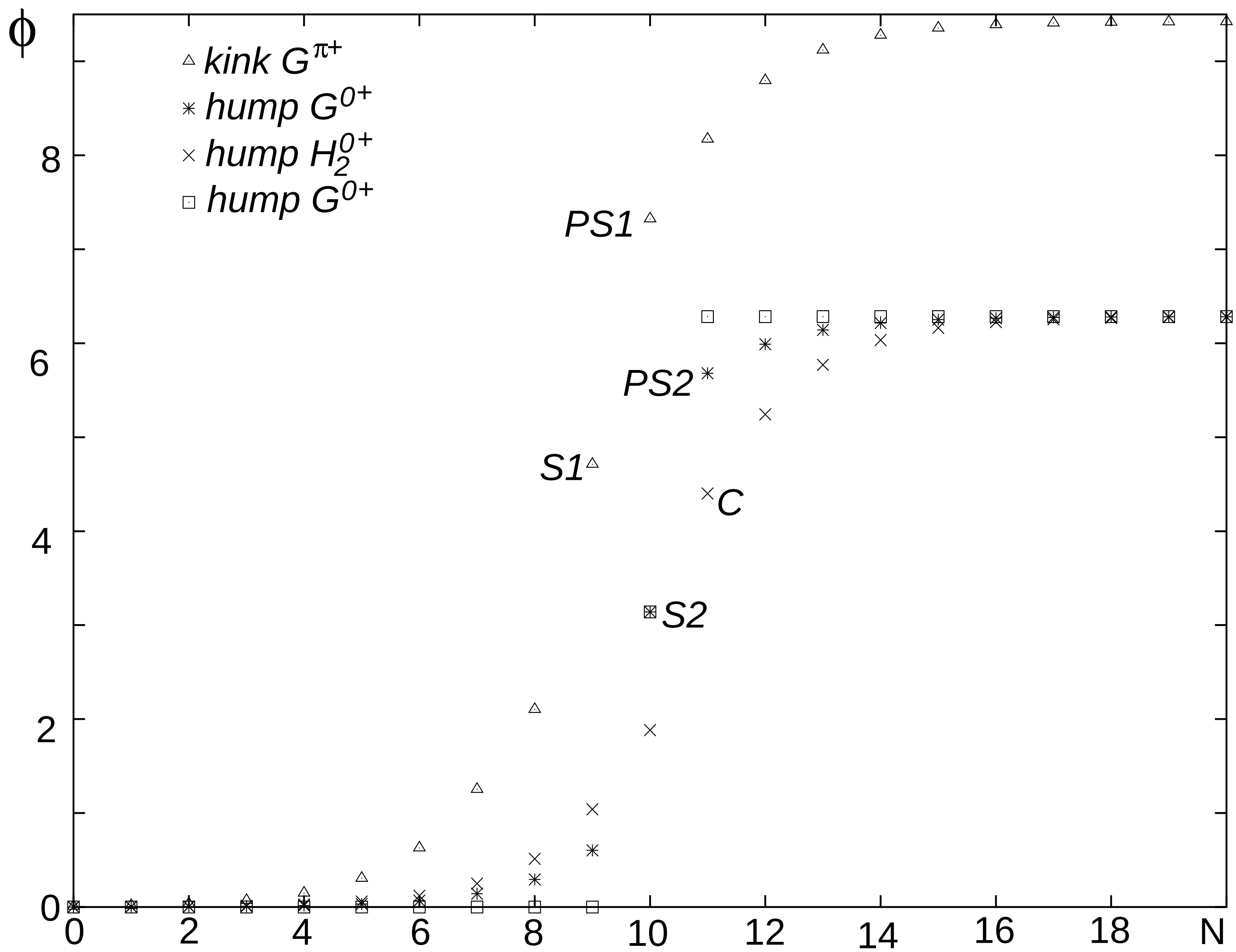

φ
kink $G^{\pi+}$
hump G^{0+}
hump H_2^{0+}
hump G^{0+}
PS1
PS2
S1
C
S2
0
2
4
6
8
0
2
4
6
8
10
12
14
16
18
N

figure 3